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NOTE

Ionization of hydrogen atom by electron impact in the presence of elliptically polarized laser field

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Abstract : The problem of ionization in electron hydrogen atom collision in the presence of elliptically polarized laser field, is investigated. The use of the higher order modification of the atomic bound state wave function shows that there is a strong enhancement in the cross section when the laser frequency is half the atomic transition frequency. The dependence of the cross section on the polarization of the laser field is also discussed.

Keywords : Hydrogen atom, electron impact ionization, elliptically polarized laser field.

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We study the problem of ionization in electron hydrogen atom collision in the presence of laser field when the electromagnetic field is elliptically polarized and obtain the results for plane polarized and circularly polarized electromagnetic field as special cases. The energy of the ejected electron can be obtained from both the projectile electron and laser field by absorption of a number of photons. In our analysis [1], we consider higher order terms (involving different unperturbed quantum states $\phi_K^{(0)}$,K being the quantum number, of the laser modified atomic wave function in the initial state given by

$$\psi(r,t) = \exp\left[(ih)^{-1}\left\{E_0^{(0)}\, t + \int_{-\infty}^{t} Re\Delta E(t)dt\right\}\right] \Phi(r,t) \tag{1}$$

where $E_0^{(0)}$ is the unperturbed ground state energy and $\int_{-\infty}^{t} Re\Delta E(t)dt$ gives the energy shift and the oscillatory phase terms.

In the above expression

$$\Phi(r,t) = \Phi_0^{(0)} + (b_K^{(1,1)}\, e^{i\omega t} + b_K^{(1,-1)}\, e^{-i\omega t}\, \Phi_K^{(0)} + \sum_{l}^{2,-2,0} (b_K^{(2,l)} + \beta_0^{(2,l)}\, \delta_{K0}\, e^{il\omega t}\, \Phi_K^{(0)} + \tag{2}$$

$$= \Phi_{O}^{(O)} + \left\{ -\frac{1}{\omega_{KO} + \omega} \right\} M_{KO}^{(1,1)} \phi_{K}^{(O)} e^{i\omega t} +$$

$$+ \left\{ -\frac{1}{\omega_{KO} - \omega} \right\} M_{KO}^{1,1} \phi_{K}^{(O)} e^{-i\omega t} + \left[\left\{ -\frac{1}{\omega_{KO} + 2\omega} \right\} M_{KK'}^{1,1} \right.$$

$$\cdot \left\{ -\frac{1}{\omega_{K'O} + \omega} \right\} M_{K'O}^{11} \Phi_{K}^{(O)} - \frac{1}{2} \left\{ \frac{-1}{\omega_{KO} - \omega} M_{KO}^{1,1} \right\}^{*}$$

$$\left. \left\{ \frac{-M_{KO}^{1,1}}{\omega_{KO} - \omega} \right\} \Phi_{O}^{(O)} \right] e^{i2\omega t} + \left[\left\{ \frac{-1}{\omega_{KO} - 2\omega} \right\} M_{KK'}^{1,1} \right.$$

$$\left. \cdot \left\{ \frac{-1}{\omega_{K'O} - \omega} \right\} M_{K'O}^{1,1} \Phi_{K}^{(O)} - \frac{1}{2} \left\{ \frac{-M_{KO}^{1,1}}{\omega_{KO} + \omega} \right\}^{*} \left\{ \frac{-M_{KO}^{1,-1}}{\omega_{KO} - \omega} \right\} \Phi_{O}^{(O)} \right]$$

$$\cdot e^{-i2\omega t} + \left[\left\{ \frac{-1}{\omega_{KO}} \left(-M_{KK'}^{1,1} \frac{1}{\omega_{KO} + \omega} M_{K'O}^{1,1} - M_{KK'}^{1,1} \frac{1}{\omega_{KO} - \omega} \right. \right. \right.$$

$$\left. \left. \cdot \left(M_{K'O}^{1,-1} \right) \right\} \Phi_{K}^{(O)} - \frac{1}{2} \left\{ \frac{1}{\omega_{KO} - \omega} M_{KO}^{1,1*} \frac{1}{\omega_{KO} - \omega} M_{K'O}^{1,-1} + \right.$$

$$\left. \left. \frac{1}{\omega_{KO} + \omega} M^{1,1*} \frac{1}{\omega_{KO} + \omega} M_{KO}^{1,1} \right\} \Phi_{O}^{(O)} \right] \tag{3}$$

where, $\omega_{KO} = E_{K}^{(O)} - E_{O}^{(O)}$, $E_{nlm} = -1/2n^2$

$$M^{1,\pm 1} = 1/2\,(x\,\varepsilon_x \mp i y \varepsilon_y).$$

In this work, the electric field associated with the laser field in general is taken to be elliptically polarized and is given by

$$\varepsilon = -\frac{1}{c}\frac{\partial A}{\partial t} \tag{4}$$

$$= \hat{e}_x \varepsilon_x \cos\omega t + \hat{e}_y \varepsilon_y \sin\omega t.$$

Then the dipole type interaction used here can be written as

$$H^{(1)} = r.\,\varepsilon(t)$$

$$= M^{1,1} e^{i\omega t} + M^{1,-1} e^{-i\omega t} \tag{5}$$

Joachain *et al* [2] in their study of above mentioned ionization problem have considered only the first order modification of bound states wave function corresponding to the term

$$b_K^{1,\pm 1} = -M_K^{1,\pm 1} / (\omega_K - \omega_O \pm \omega)$$

and have observed that its contribution becomes very large when laser frequency ω is nearly equal to atomic transition frequency ω_{KO} i.e., ($\omega = \omega_K - \omega_O$). The second order modification of the bound state wave function (considered in our work) which becomes very appreciable for strong laser field, corresponding to the $b_K^{2, \pm 2}$ term containing the factor $1/(\omega_{KO} \pm 2\omega)$ clearly shows that there is a strong enhancement in the cross section even when laser frequency is half the transition frequency i.e. $1/2(\omega_K - \omega_O)$. Now very near this resonance case (where $\omega = 1/2(\omega_K - \omega_O)$ the cross section is still appreciable where the laser field is not very strong i.e. $\varepsilon < 5 \times 10^{11}\ V\, m^{-1}$ (the atomic unit of field strength) when the time dependent purturbation theory is somewhat satisfactory. We study the energy dependence and polarization dependence of the triple differential cross section (TDCS) for electron impact ionization. The Hamiltonian of the target atom in the presence of the laser field can be written as

$$H = \frac{1}{2m}\left[p + \frac{e}{c} A\ (t)\right]^2 + U(r) \tag{6}$$

where $A(t)$ is the vector potential associated with the laser field and $U(r)$ is the atomic potential.

We consider the following gauge transformation

$$\psi'(r,t) = \exp\left[-\frac{i.e\ A(t).\bar{r}}{\hbar c}\right]\psi(r,t) \tag{7}$$

$$i\frac{\partial}{\partial t}\psi(r,t) = \exp\left[-\frac{i.e.\ A(t).\ r}{\hbar c}\right]\psi(r,t) \tag{8}$$

$$i\frac{\partial}{\partial t}\psi(r,t) = (H^{(O)} + H^{(1)})\ \psi(r,t) \tag{9}$$

$$H^{(O)} = \frac{1}{2m}P^2 + U(r). \tag{10}$$

The perturbed Hamiltonian $H^{(1)}$ is given by

$$H^{(1)} = r.\varepsilon\ (t) \tag{11}$$

As the wavelength of laser field is taken to be very large compared to the dimension of the atom we ignore the space dependence of the vector potential A. This assumption leads to the dipole type interaction given above.

In the case of $(e, 2e)$ collision in the presence of a laser field, we assume that a fast electron of momentum k_i is incident on the target and a fast scattered electron of momentum k_A is detected in coincidence with a some what slow ejected atomic electron of momentum k_B. We ignore exchange effect in this problem.

The first Born approximation of S-matrix element for ionization is given by (in the notation of Joachain *et al* [2]).

$$S^{B}_{ion} = -i \int_{-\infty}^{\infty} dt <\chi_{k_A}(r_O,t), \Phi_{k_B}(r_1,t) \left| \frac{1}{r_{o1}} - \frac{1}{r_o} \right| \chi_{k_1}(r_O,t), \Phi_o(r_1,t)> \tag{12}$$

where r_O and r_1 are respectively the coordinates of the incident and target electrons and

$r_{o1} = |r_o - r_1|.$

The laser dressed wave functions $\chi_{k_1}(r_o,t)$ and $\chi_{k_A}(r_o,t)$ are of the form

$$\chi_{k'}(r,t) = (2\pi)^{-3/2} \exp\left[i\left\{ k.r - E_k{}' - (k,\alpha) \sin\left(\omega t + \delta_k + \frac{\pi}{2}\right) - \frac{e^2}{2mc^2} \int^t A^2(t')\,dt' \right\}\right]. \tag{13}$$

$$\alpha = \varepsilon / \omega^2 \tag{14}$$

where the notation (k, α) stands for

$$(k,\alpha) = \left[(k_x \alpha_x)^2 + (k_y \alpha_y)^2 \right]^{1/2} \tag{15}$$

and

$$\delta_k = -\tan^{-1} \frac{k_y \alpha_y}{k_x \alpha_x} \tag{16}$$

For plane-polarized light of the type $\varepsilon_x = 0$ and $\varepsilon_y \neq 0$,.(k,α) becomes $k_y \alpha_y = k.\alpha$ say) and

$$\delta_K + \frac{\pi}{2} = 0. \tag{17}$$

For the circularly polarized light for which $\varepsilon_x = \varepsilon^c$ and $\varepsilon_y = \pm \varepsilon_x$, the term (k,α) becomes $k\alpha^c$ and δ_k has the value $\mp \tan^{-1}(k_y/k_x)$.

We can then write

$$\chi_k = (2\pi)^{-3/2} \exp\left[i(k.r - E_k t - \frac{e^2}{2mc^2} \int^t A^2(t')\,dt') \right.$$

$$\left. \cdot \Sigma J_l \{(k,\alpha)\} \exp\left[-il\left(\omega t + \delta_k + \frac{\pi}{2}\right)\right]\right]. \tag{18}$$

The dressed wave function $\Phi_O(r,t)$ for the initial bound state has already been given. For high values of k_A and k_B or high energy free electrons considered in this paper the laser modified continuum wave function $\phi_{k_B}(r,t)$ of the ionized electron of momentum k_B can be fairly approximated by the laser dressed plane wave function $\chi_{k_b}(r,t)$ defined by relation like (18). For quite slow electrons we have to consider laser dressed Coulomb wave function as determined by Basile *et al* [3] and used by Joachain *et al* [2] and Basile *et al* [3] Using the above [2] approximation we obtain from eqs. (12), (13) and (18) the following expression for S matrix element.

$$S^{B}_{\text{ion}} = (2\pi)^{-1} i \sum_{l=-\infty}^{\infty} \delta(E_{k_A} + E_{k_B} - E_{k_i} - E_0 - l\omega) f^{B,l}_{\text{ion}} \tag{19}$$

where $f^{B,l}_{\text{ion}}$ is the first Born approximation to the $(e, 2e)$ collision amplitude with the transfer of l photons and is given by

$$f^{B,l}_{\text{ion}} = \left(f^{l}_{I} + f^{l}_{II} + f^{(2\omega),l}\right) \exp\left[-il(\delta_q + \tfrac{\pi}{2})\right] \tag{20}$$

where

$$f^{l}_{I} = -2K^{-2} \langle \chi_{K_B} | exp(iK.r) | \psi_0 \rangle$$

$$f^{l}_{II} = iK^{-2} \sum_{nlm} \langle \chi_{k_B} | \exp(iK.r) | \psi_{nlm} \rangle \left[\frac{J_{l-l}(\lambda) M^{1,1}_{nlm,0}\, e^{i(\delta_q + \frac{\pi}{2})}}{E_n - E_0 - \omega} \right. \tag{21}$$

$$\left. - \frac{J_{l+l}(\lambda) M^{1,-1}_{nlm,0}\, e^{-i(\delta_q + \frac{\pi}{2})}}{E_n - E_0 + \omega} \right] \tag{22}$$

and

$$K = k_i - k_A . \tag{23}$$

The momentum transferred to the ionized atom is

$$Q = K - k_B \tag{24}$$

$$\lambda = Q.\, \alpha_0. \tag{25}$$

Retaining those terms of second order in laser field strength of the dressed wave function $\Phi_0(r,t)$ which dominate when $2\omega \approx E_n - E_0$ $(n = 2, 3, ...)$ we can write

$$f^{(2\omega),l} = K^{-2} \sum_{K} \sum_{K'} \frac{1}{\omega_{KO} - 2\omega} M_{KK'} \frac{1}{\omega_{K'O} - \omega} M_{K'O} J_{l-2}(\lambda)$$

$$\langle \psi_{k_b} | e^{iK.r} | \Phi^{(0)}_{k} \rangle \tag{26}$$

The triple differential cross section for impact ionization is given by

$$\frac{d^3\sigma^{B,l}_{\text{ion}}}{d\Omega_A d\Omega_B dE} = \frac{k_A k_B}{k_i} |f^{B,l}_{\text{ion}}|^2 \tag{27}$$

From eqs. (21) and (22) we have

$$f^{l}_{I} = -2K^{-2} J_l(\lambda)\, (2\pi)^{-3/2} \int \exp\left[i(K - k_B)\right] r\, N_{1o}\, e^{-r\beta} \frac{1}{\sqrt{4\pi}}\, dr$$

$$= -2K^{-2} J_l(\lambda) \sqrt{2}\ (2\pi)^{-1} \left(-\frac{\delta}{\delta\beta}\right) \frac{1}{\beta^2 + Q^2} \Bigg|_{\beta = 1} \tag{28}$$

and

$$f_{II}^{l} = 2iK^{-2} (2\pi)^{-1/2} \sum_{n=2,3,} N_{nl} I_{nl}^{*} \left[R'_{nl} \left(\frac{-\partial}{\partial\beta}\right) \frac{2(\varepsilon, Q)}{-(\beta^2 + Q^2)^2} \right]$$
$$\left[\frac{J_{l\,l}(\lambda)}{E_n - E_O - \omega} - \frac{J_{l+1}(\lambda)}{E_n - E_O + \omega} \right] \tag{29}$$

where $I_{nn'}$ is defined by the relation

$$I_{nn'} = \int r\, \psi^{*}_{n'l.m}\, \psi_{noo}\, Y_{lm}\, dr \tag{30}$$

and we have taken the hydrogenic wave function in the form

$$\psi_{nlm} = N_{nl}\, e^{-r/n} R_{nl}(r)\, Y_{lm}(\hat{r}) = R_{nl}(r)\, Y_{lm}(\hat{r}) \tag{31}$$

In the evaluation of $f^{(2\omega),l}$ we assume that most of the contribution comes when K corresponds to the $2s$ and $3s$ state and K' to the $2p$ state.

$$f^{(2\omega),l} = 2K^{2} \sum_{n=2,3,..} \sum_{n'=2,3,} \frac{1}{E_n - E_O - 2\omega} \frac{1}{E_n - E_O - \omega} J_{l-2}(\lambda)\, e^{i2(\delta_Q + \pi/2)}$$
$$I^{*}_{n'n} I^{*}_{n'l} \left(\varepsilon_x^2 - \varepsilon_y^2\right) (2\pi)^{3/2} N_{no} \cdot$$
$$\cdot \frac{4\pi}{\sqrt{4\pi}} \left[R'_{no} (-\partial/\partial\beta) \frac{1}{Q^2 + \beta^2} \right] \Bigg|_{\beta = 1/n} + \ldots\ldots\ldots \tag{32}$$

The terms f_I^l, f_{II}^l and $f^{(2\omega),l}$ of the scattering amplitude has different dependence on polarization vector of laser field as described below.

(1). For plane-polarized laser (of the type described after eq. (17) f_I^l vanishes for $l \neq O$ when ε is perpendicular to Q i.e. ($\varepsilon \perp Q$) but f_{II}^l vanishes for any value of l. In the above cases $f^{(2\omega),l}$ for plane polarized light still remains nonvanishing for $l = 2$ because of the presence of the factor ε_y^2 in the expression for $f^{(2\omega),l}$. In this case, we should keep in mind that for plane polarized light of the type described after eq. (17), the ($\varepsilon_x^2 - \varepsilon_y^2$) term occurring in eq. (32) is to be replaced by $-\varepsilon_y^2$ term. (2) For the particular case of circularly polarized laser i.e. $\varepsilon_x^2 = \varepsilon_y^2$, the dominant part of the term $f^{(2\omega),l}$ always vanishes unlike the f_I^l and f_{II}^l terms.

In Figures 1 and 2 we plot the triple differential cross section (TDCS) for electronimpact ionization versus energy ω of plane polarized laser photon (($\varepsilon_O \parallel k_i$) for the field strength. 0.02 a.u. and 0.0635 a.u. respectively. The energy of the incident electron is

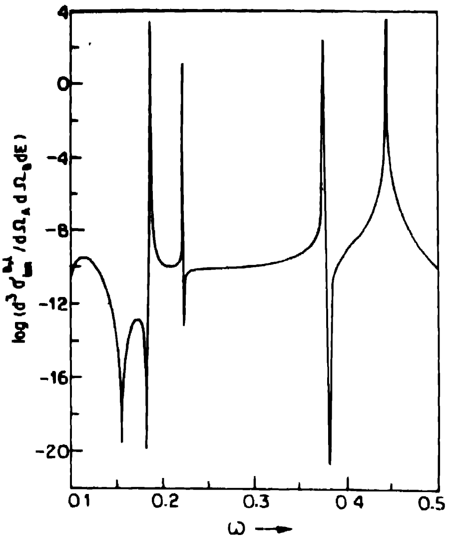

Figure 1. Triple differential cross section (evaluated in atomic units) for electron impact ionization in the presence of plane polarized laser versus photon energy ω for laser field strength 0.02 a.u.

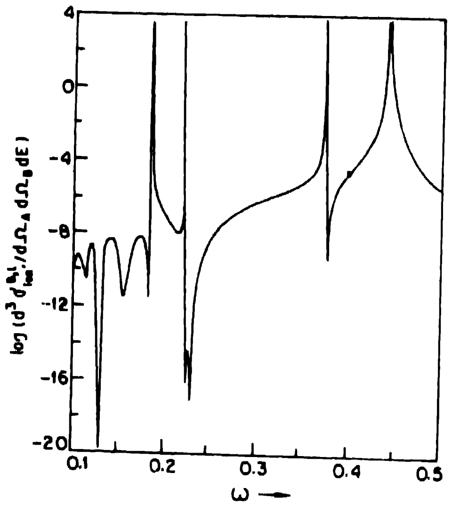

Figure 2. The same as in Figure 1 but for laser field strength 0.0635 a.u.

taken to be 20 a.u. and that of the scattered electron to be 10 a.u. The momenta of both the scattered electron and the ejected electron are taken to be parallel to the projectile electron momentum in our calculation. All the quantities appearing in the Figures 1 and 2 are evaluated in atomic units. It appears that when twice the energy of the laser photon (ω) is nearly equal to the energy difference between $2s$ state and ground state $E_2 - E_0 (\approx 2\omega)$ (i.e. ω = 0.1875 a.u.) or that between $3s$ state and ground state $E_3 - E_0 (\approx 2\omega)$ (i.e. ω =0.222 a.u.) there is a sharp rise in the differential cross section. This also holds in general whenever 2ω equals $E_n - E_0 (\approx 2\omega)$ for any values of n (principal quantum number). In the above case $(2\omega \approx E_n - E_0)$, $f^{(2\omega),I}$ term obviously dominates over f^{I}_{I} term and f^{I}_{II} term. We may note that f^{I}_{II} and $f^{(2\omega),I}$ terms arise due to the first order and second order laser modification of the ground state of hydrogen atom respectively. We also find usual class of resonances arising from f^{I}_{II} term whenever $\omega \approx E_n - E_0$ and in particular for ω = 0.375 a.u., 0.444 a.u., ... In Figure 2 we further find more oscillations than in Figure 1 and this may be due to the fact that the argument λ of $J_l(\lambda)$ is greater in Figure 2 than in Figure 1 for the same value of ω. It may be noted that the strength of $(2\omega \approx E_n - E_0)$ class of resonance is second order in laser field strength where as the strength of $(\omega \approx E_n - E_0)$ class of resonance is first order in field strength.

For other values of ω and arbitrary direction of Q and ε, the term f_I^l involving $J_l(\lambda)$ factor can be made almost to vanish by choosing appropriate field strength of the laser field. In this case also the f_{II}^l term and $f^{(2\omega),l}$ dominate over the f_I^l term.

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